Introduction to system models

Definition

- A system
  - a set of interacting components grouped together in order to study some part of the real world
- A mathematical model
  - A simplified mathematical representation of the relationship between variables
- A (mathematical) system model
  - A representation of a system in equations
  - Usually dynamic equations to describe the dynamics of the system

An example

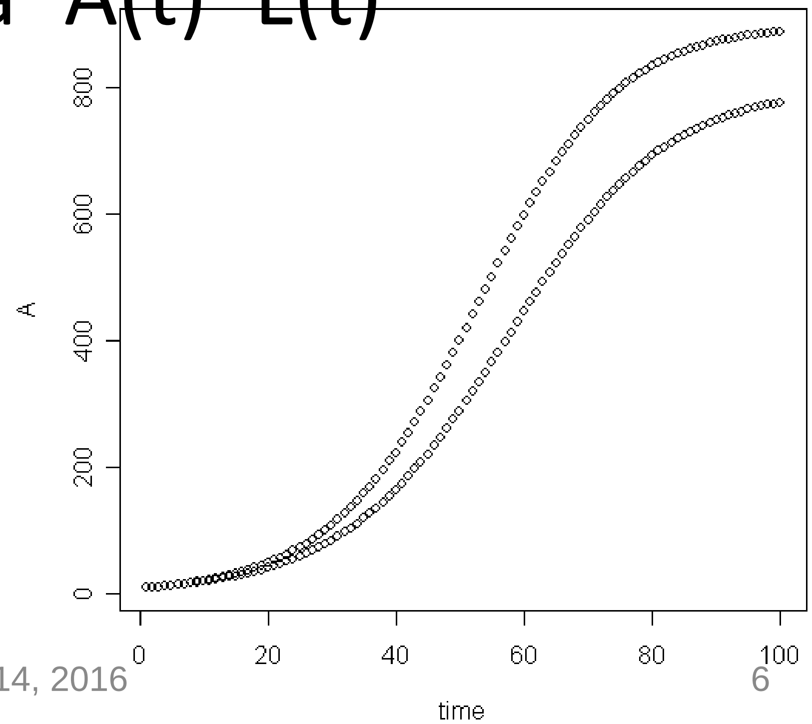
predator prey interaction

Predator prey model

- $A(t)$=number of individuals of prey at time t ($A(t)$=aphids at time t)
- $L(t)$=number of individuals of predator at time t ($L(t)$=ladybugs at time t)

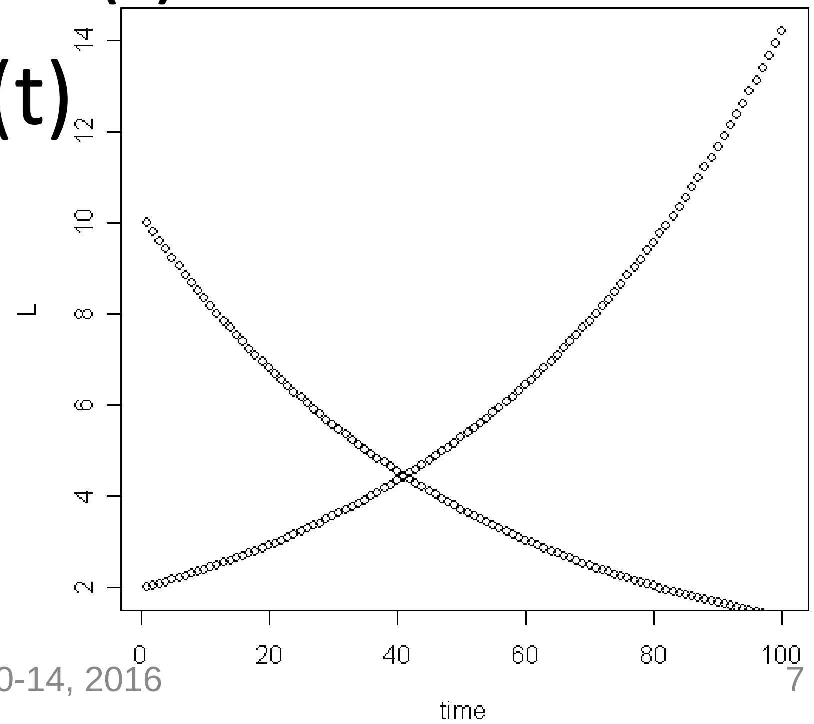
Model for prey (for given number of predators)

- Prey have logistic growth, plus mortality due to predation
- Predation proportional to A(t) and to L(t)
- $dA/dt=r_A*A(t)*(1-A(t)/K)-a*A(t)*L(t)$



Model for predator (for given number of prey)

- Increase depends on food supply, proportional to A(t) and to L(t)
- Mortality is proportional to L(t)
- $dL(t)/dt = b*A(t)*L(t) - r_L*L(t)$



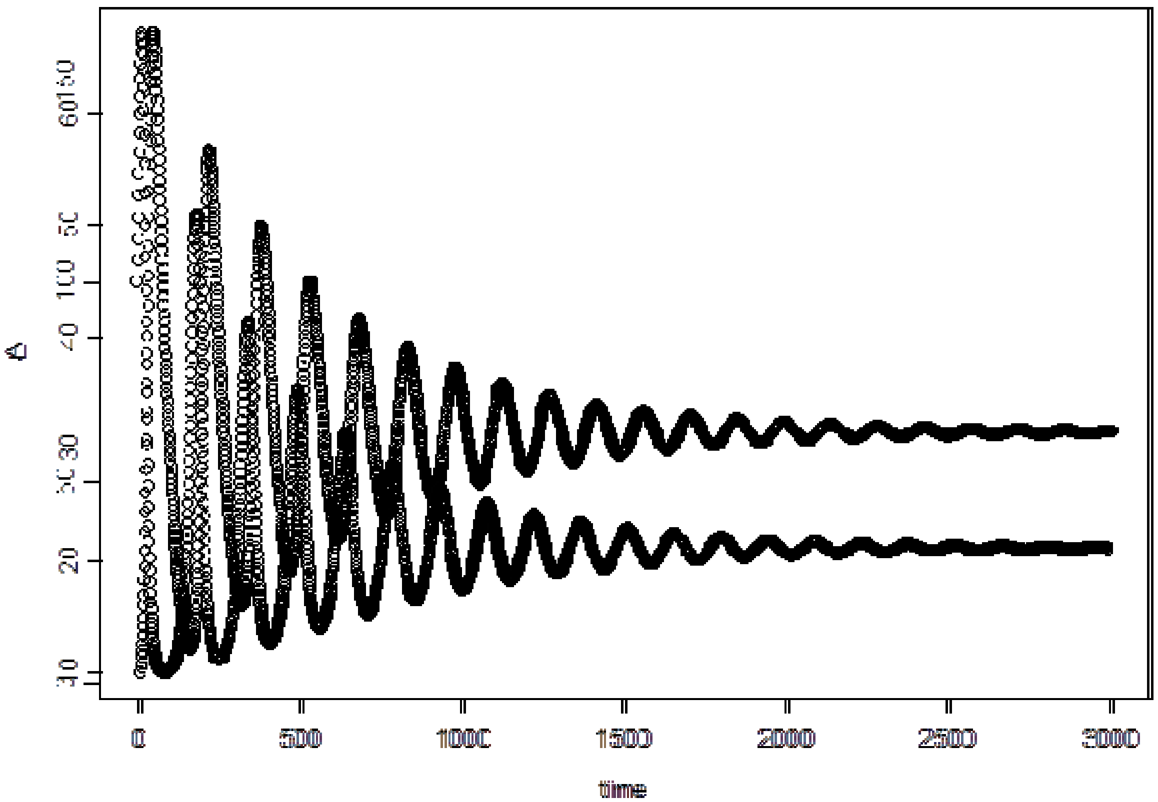
Predator and prey together

- Previously
  - Prey dynamics for fixed number of predators
  - Predator dynamics for fixed number of prey
- Now include interaction.
  - Solve equations simultaneously
  - This is a (dynamic) system model (dynamic equations for two components, A and L, that interact)

$$dA(t)/dt=r_A*A(t)*(1-A(t)/K)-a*A(t)*L(t)$$

$$dL(t)/dt= b*A(t)*L(t)– r_L*L(t)$$

Predator and prey together



Out-of-phase oscillations.

Importance of systems approach

- Study of components is necessary but not sufficient to understand system behavior
- The system can have behavior that you don’t see by studying individual components.

Elements of a dynamic system model

State variables

- These are the variables whose dynamics are calculated
  - In our case A(t), L(t)
- Easily recognized, they appear on left of equations, as d(state variable)/dt

$$dA(t)/dt=r_A*A(t)*(1-A(t)/K)-a*A(t)*L(t)$$

$$dL(t)/dt= b*A(t)*L(t) – r_L*L(t)$$

– The choice of state variables is very important

– That defines what is included in the system

  - We only calculate the dynamics of aphids and ladybugs, not other populations, or plants etc. The choice of state variables is very important

Explanatory variables

- These are measurable variables that affect dynamics of system, but aren't affected by system.
  - Appear only on right of equations, plus initial values
  - Here, only the initial values A(t=0) and L(t=0)
  - No time dependent explanatory variables
    - Could have temperature for example

$$dA(t)/dt=r_A*A(t)*(1-A(t)/K)-a*A(t)*L(t)$$

$$dL(t)/dt= b*A(t)*L(t)– r_L*L(t)$$

- The choice of explanatory variables is very important

  1. Because to simulate with the model, we need to know those values

     – This determines whether model can be used in practice. If we don't know A(t=0) and L(t=0), can't simulate dynamics of system

  2. Because it sets an upper limit on how good the model can be

     - Here, no weather variables, no plant variables, no variables for other populations. The model can't describe variability due to ignored explanatory variables.

Parameters

- Not calculated by the model and not measured for each field.
- Here there are five: r_A, K, a, b, r_L

$$dA(t)/dt = r_A * A(t) * (1 - A(t)/K) - a * A(t) * L(t)$$

$$dL(t)/dt = b * A(t) * L(t) - r_L * L(t)$$

- Where do the values of the parameters come from?
  - Not calculated by the model, not measured
    - They come from past studies of similar systems.
    - We can use past values because we assume parameters don't change. Same values for all fields (or some subset of fields).

- The estimation of parameter values is very important
  - Different values can give very different predictions.
  - This is a major difficulty in modeling (system or other)

Output variables

- Often, not interested in full dynamics, but in some specific output.
  - e.g. yield of a crop, integrated damage of a disease like $\int A(t)dt$
  - In that case, we can write the model as $\hat{y}=f(X;\theta)$
    - $\hat{y}$ is simulated output. It is calculated as function of explanatory variables X and parameters θ.

- It is important to define the outputs of interest.
  - Quality of model can be very different for different outputs.

What is special about system models?

- If we look at one output, a system model is just a regression model
  - Relates output to explanatory variables
  - Standard regression equation is $Y=f(X;\theta)+\varepsilon$
  - Observed Y is simulated value plus error
  - So what’s special?

Specificity of system models

- We have two complementary ways of studying dynamic system models
  - We can study individual processes
  - We can study the overall system
  - This is a major difference with other models

1. We can study the individual processes
   - rate of increase of aphids in absence of ladybugs
   - rate of predation versus density of aphids
   - natural mortality of ladybugs
   - effect of predation rate on rate of increase of ladybugs
2. We can study full system
   - Aphids and ladybugs interacting.

- Those involve quite different experiments
- A major challenge in system modeling is combining these two sources of information.
  - – In particular, we have two ways of getting the parameter values
    - We can estimate them either by studying the individual processes or by studing the overall system

END