



Sensitivity and uncertainty analyses

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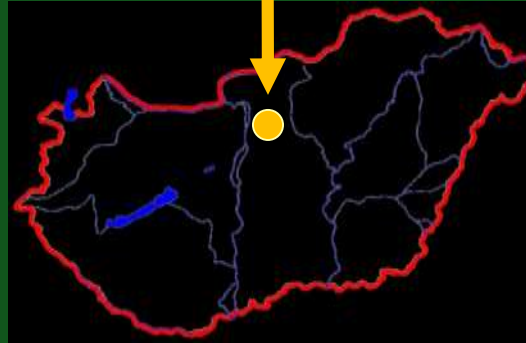
ENDURE Summer School 2016

The role of IPM in mitigating pest development under climate change—modelling approaches

Hungary



Gödöllő



Szent István University



Desired modelling result

Clear answer(s)

Considering X the result will be Y

Your model will be uncertain

= variability in the output

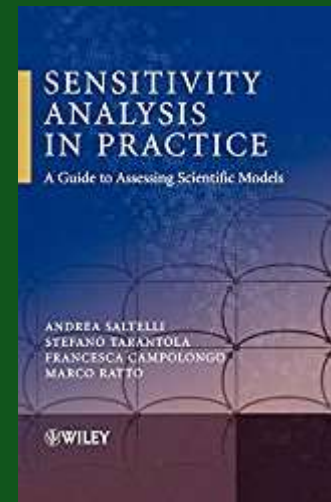
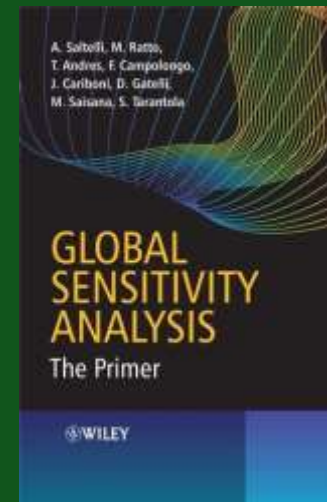
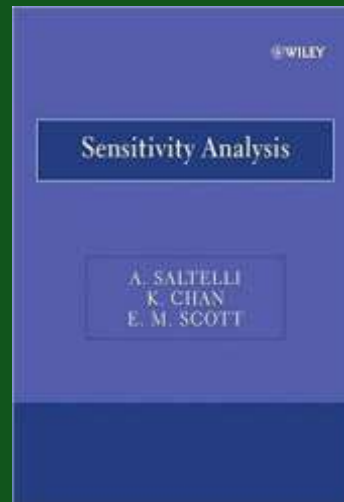
Terminology

What is sensitivity analysis?

and what is uncertainty analysis?

Disclaimer

Andrea Saltelli



Disclaimer

https://en.wikipedia.org/wiki/Sensitivity_analysis

WIKIPEDIA The Free Encyclopedia

Sensitivity analysis

From Wikipedia, the free encyclopedia

Sensitivity analysis can refer to two approaches of analyzing results of a **mathematical model** or system (numerical or otherwise).

- One approach addresses the question of "What's important to model or system development?" One can seek to identify important connections between observations, model inputs, and predictions or forecasts. That is, one can seek to understand what observations (measurements of dependent variables) are most and least important to model inputs (parameters representing system characteristics or excitation), what model inputs are most and least important to predictions or forecasts, and what observations are most and least important to the predictions and forecasts. Often the results are surprising, lead to finding problems in the data or model development, and fixing the problems. This leads to better models. For more information about this kind of sensitivity analysis see ^{[1][2]}
- The second approach addresses the question "What's important to calculated measures of uncertainty?". This is the study of how the *uncertainty* in the output of a **mathematical model** or system (numerical or otherwise) can be apportioned to different sources of uncertainty in its inputs ^{[2][4]} A related practice is **uncertainty analysis**, which has a greater focus on uncertainty quantification and propagation of uncertainty; ideally, uncertainty and sensitivity analysis should be run in tandem.

The remainder of this article focuses on the second approach to sensitivity analysis.

Taking an example from economics, in any budgeting process there are always variables that are uncertain. Future tax rates, interest rates, inflation rates, headcount, operating expenses and other variables may not be known with great precision. Sensitivity analysis answers the question, "if these deviate from expectations, what will the effect be (on the business, model, system, or whatever is being analyzed), and which variables are causing the largest deviations?"

The process of recalculating outcomes under alternative assumptions to determine the impact of a variable under sensitivity analysis can be useful for a range of purposes,^[3] including:

- Testing the **robustness** of the results of a model or system in the presence of uncertainty
- Increased understanding of the relationships between input and output variables in a system or model.
- Uncertainty reduction, through the identification of model inputs that cause significant uncertainty in the output and should therefore be the focus of attention in order to increase robustness (perhaps by further research).
- Searching for errors in the model (by encountering unexpected relationships between inputs and outputs).
- Model simplification – fixing model inputs that have no effect on the output, or identifying and removing redundant parts of the model structure.
- Enhancing communication from modelers to decision makers (e.g. by making recommendations more credible, understandable, compelling or persuasive).
- Finding regions in the space of input factors for which the model output is either maximum or minimum or meets some optimum criterion (see *optimization* and Monte Carlo filtering).
- In case of calibrating models with large number of parameters, a primary sensitivity test can ease the calibration stage by focusing on the sensitive parameters. Not knowing the sensitivity of parameters can result in time being uselessly spent on non-sensitive ones.^[6]

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 - 3.2 Local methods
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 - 3.4 Regression analysis

Terminology

Sensitivity analysis (investigating uncertainty to answer)

What's important to model or system development?

What's important to calculated measures of uncertainty?

uncertainty analysis

Sources of uncertainty

1. Some aspects of the system are not exactly known

Weak point(s) of model structure

Garbage in garbage out?

Remember the model ensembles (Daniel)

2. Uncertainty in the input

Inputs: parameters and explanatory variables

What to do?

[Activity]

Input?

Literature, spread of a pest :

input of 3 and 4 (km/year)

The number of generations:

input of 3 and 4

Investigate the input space! (What is input space?)

What to do?

[Activity]

Investigate the input space! HOW?

Brute force method:

run the model with all possible inputs

It is often time consuming (CPU time, cost)

SAMPLE the input space!

Note

It is often time consuming (CPU time, cost)
advise can be: do a more efficient code

It is better to code in 3 hours and run it
during your lunch break than code it in 3
days and run it in 10 minutes

What to do?

[Activity]

Investigate the input space! HOW?

Brute force method:

run the model with all possible inputs

It is often time consuming (CPU time, cost)

SAMPLE the input space!

How to sample the input space?

[Activity] [in pairs?]

two input variables, good sampling

(Remember: we want to do sensitivity analysis)

Problems of one-at-a-time (OAT)

Global SA: the entire input space is interesting

Local SA?

sometimes it is also interesting, but

Global SA - sampling

We can be happy... running our model not once not twice but...

Sampling often means randomness

pseudorandom...

'true randomness'

www.random.org

randomness comes from atmospheric noise

Global SA - sampling

Discrepancy

-R

our desire: a sequence fills the input space
leaving no gaps

plot

cost effective?

```
N=50
dimen=8
x.rand=matrix(runif(N*dimen),
              nrow=N,ncol=dimen)
par(mfrow=c(2,2))
plot(x.rand[,3:4])
plot(x.rand[,5:6])
plot(x.rand[,7:8])
plot(x.rand[,4:5])
par(mfrow=c(1,1))
```


Quasi random sampling

Low-discrepancy sequences
are also called quasi-random or
sub-random sequences

Quasi random sampling

Sobol sequence ☺

randtoolbox package --R

```
N=50
dimen=8
x.sob=sobol(N,dimn)
x.rand=matrix(runif(N*dimen),
              nrow=N)

par(mfrow=c(2,2))
plot(x.sob[,5:6])
plot(x.rand[,5:6])
plot(x.sob[,3:4])
plot(x.rand[,3:4])
par(mfrow=c(1,1))
```

Quasi random sampling

Sobol sequence ☺

randtoolbox package --R

hint: correlation in the Sobol sequence

If you have N input factors, generate $N+2$ dimensions and drop the first two

Quasi random sampling

Sobol sequence 😊, but it is $[0,1]$

How to translate this to the real inputs?

Can I use it for categories such as scenario RCP 2.6, RCP 4.5?

Uniform distribution vs non-uniform

Global SA

Problems of one-at-a-time (OAT) approach

We have a nice (Sobol) sequence

That means we first have the inputs

then run the model several times

then ??? Where does SA come???

Global SA

we first have the inputs

then run the model several times

then ??? Where does SA come???

Output?

Lets's assume

the output is a single numerical value

Global SA

we first have the inputs

then run the model several times

then calculate sensitivity metrics

variance based SA

variance decomposition

ANOVA-like

!!! independent inputs !!! e.g. % of maize, % of cont. maize

Variance based global SA

Main effect index (or "first-order sensitivity index"):

effect of varying X_i alone, but averaged over variations in other input parameters.

Total-effect index:

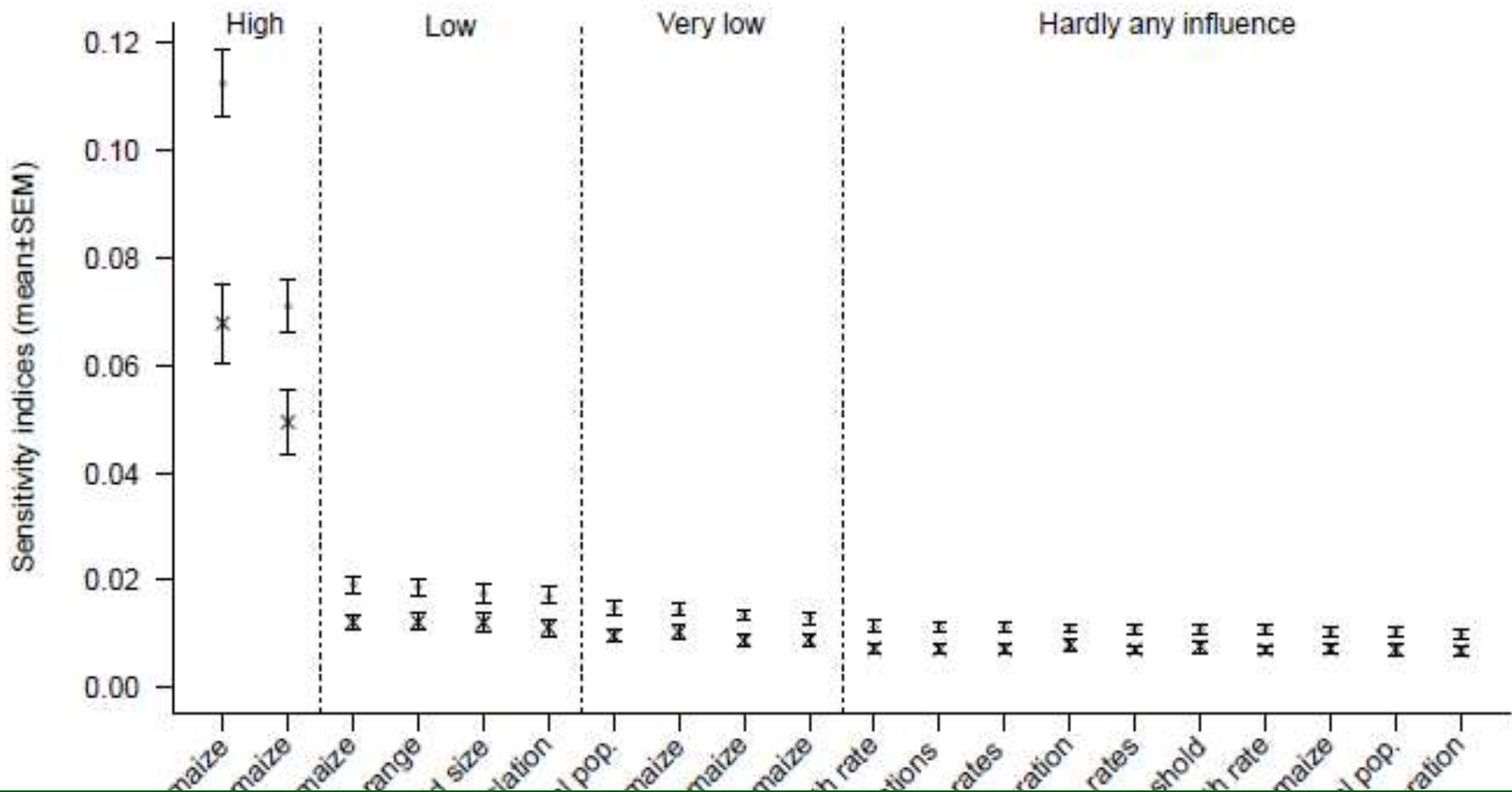
contribution to the output variance of X_i , including all variance caused by its interactions

Variance based global SA

Often

There will be a few influential input

Many inputs have hardly any effect on the output



Variance based global SA

Often

There will be a few influential input

Many inputs have hardly any effect on the output

Simplify?

Where to do more effort?

Global SA

Problem: Too many variables

Variance based method still costs a lot
try to simplify (always can be suggested by
others)

ok this is the fruit of your work

screening to identify non-influential inputs

Morris method (this is basically a OAT-way)

Desired modelling result

Clear answer(s)

Considering X the result will be Y

BUT your model will be uncertain

= variability in the output

**How to communicate this
to non-modellers?**

How to communicate this to non-modellers?

Plots - simple plots



particularly: `ggplot2`

metamodels (emulators) as you have only a few really influential inputs

How to communicate this to non-modellers?

Plots - simple plots

Interactive plots? Shiny?

