Video 7. Cross validation

MODULE 8 Model evaluation

# How to estimate MSEP if sample is used for calibration

The situation

- We have data from the target population.
- We want to use those data to estimate parameters
- But we also want to estimate MSEP.
- But if we use data for calibration, then MSE is not a good estimator of MSEP.
- What should we do?

Data splitting

- Divide data into 2 parts, one part for calibration, one part to estimate MSEP.
  - The second part isn't used for calibration, so MSE estimates MSEP.
 - (Assuming it is random sample from target population)

All the data
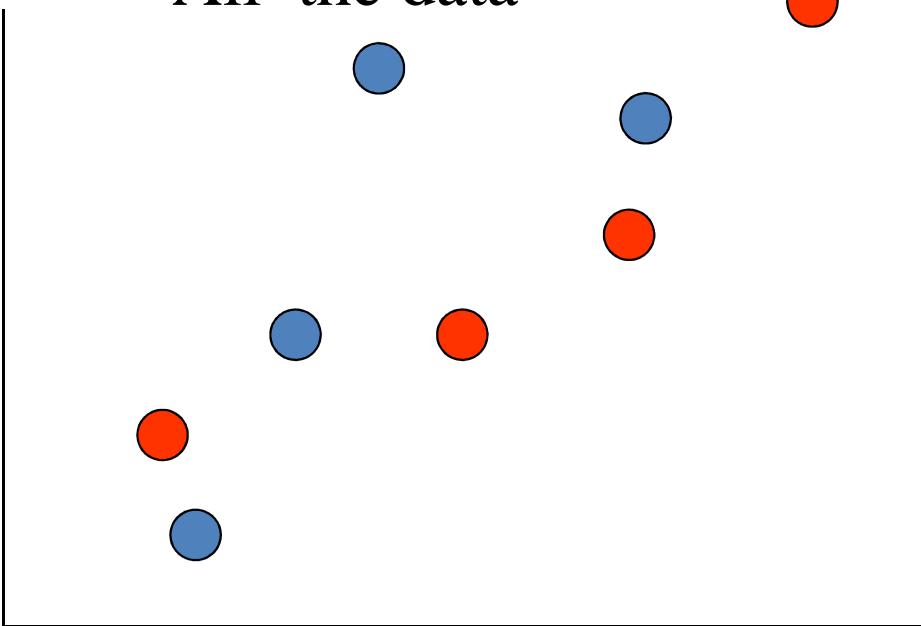
Some for
calibration
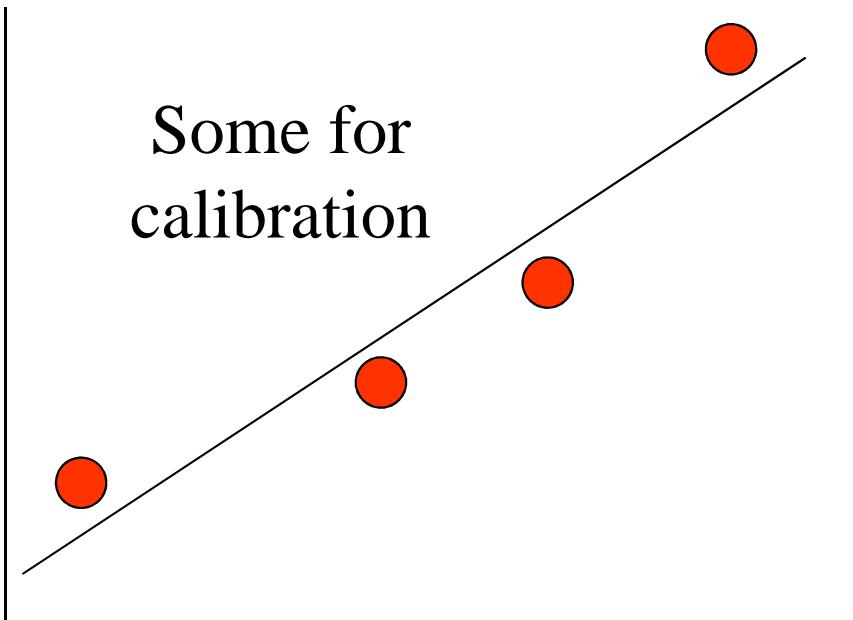
Some to estimate
MSEP

5
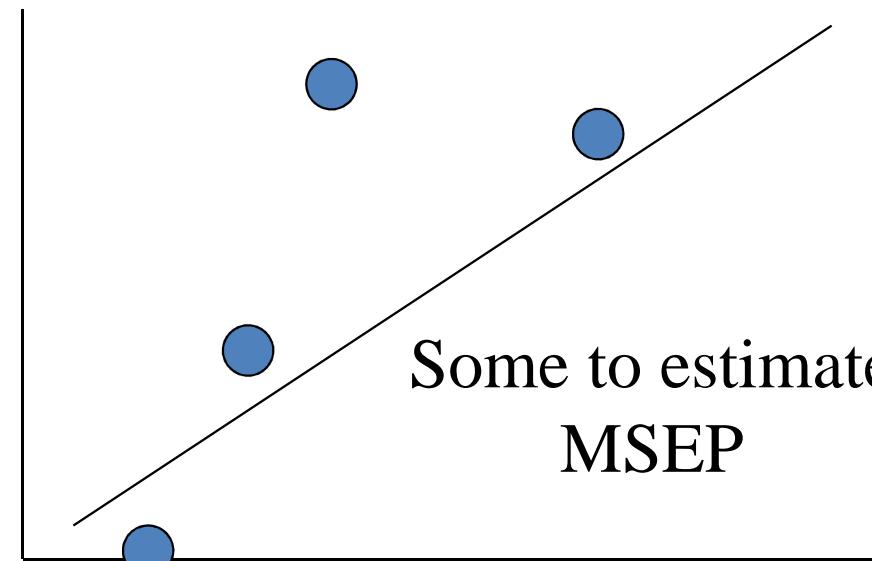
But….

- The division is arbitrary
- Calibration suffers (not all data)
- Estimation of MSEP suffers (not all data)

why this?

and not this?
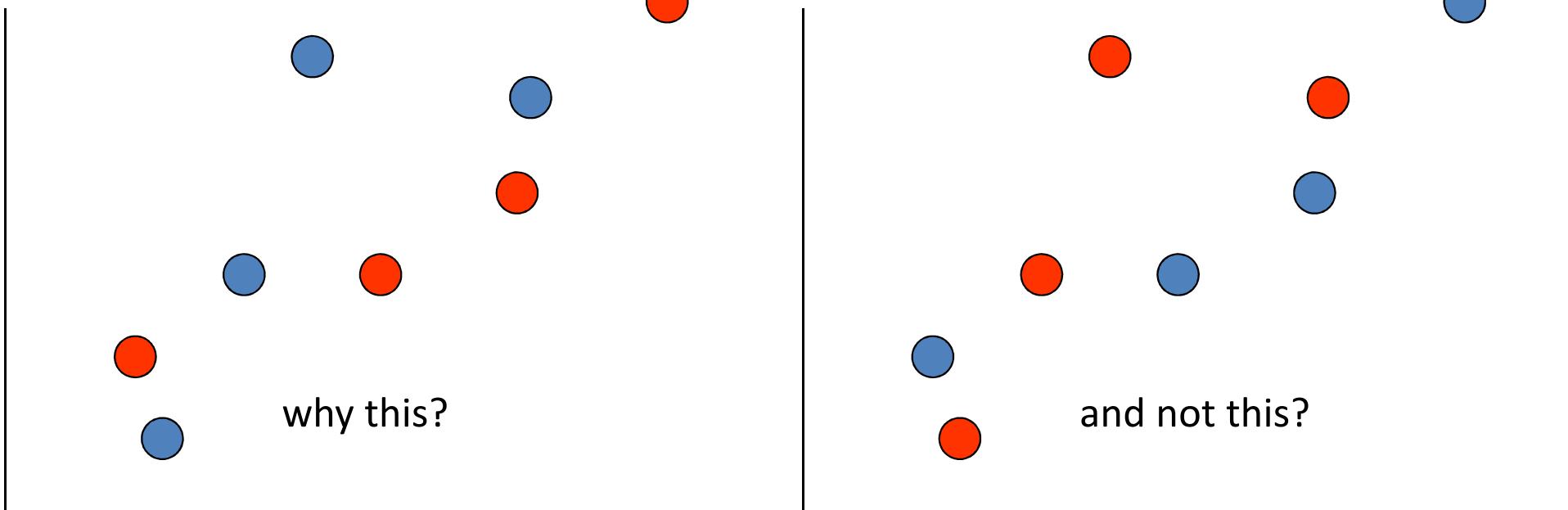
Cross validation

- A more efficient way to use the data.

- For N data points, do N different divisions of data.

- In each division, calibrate with N-1 data points, estimate MSEP with one data point.
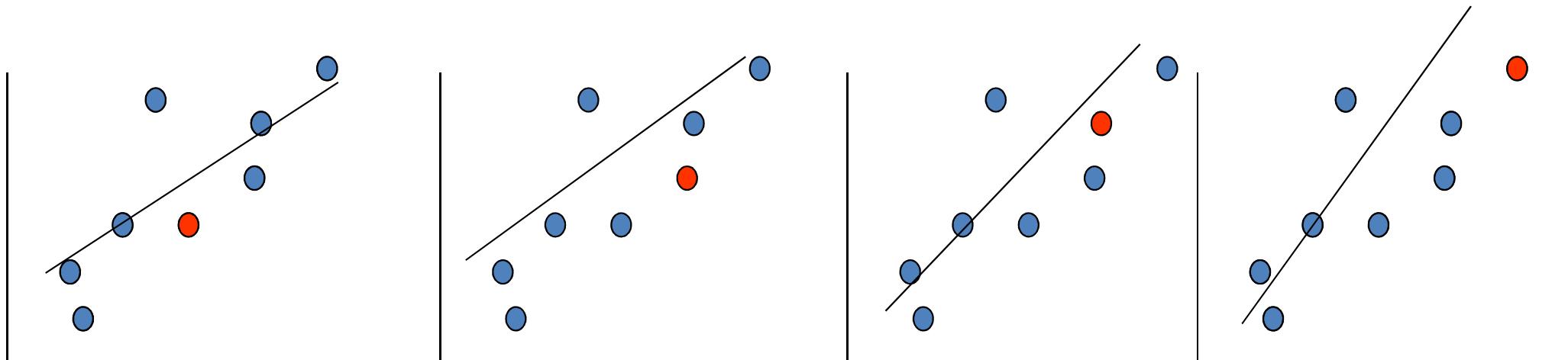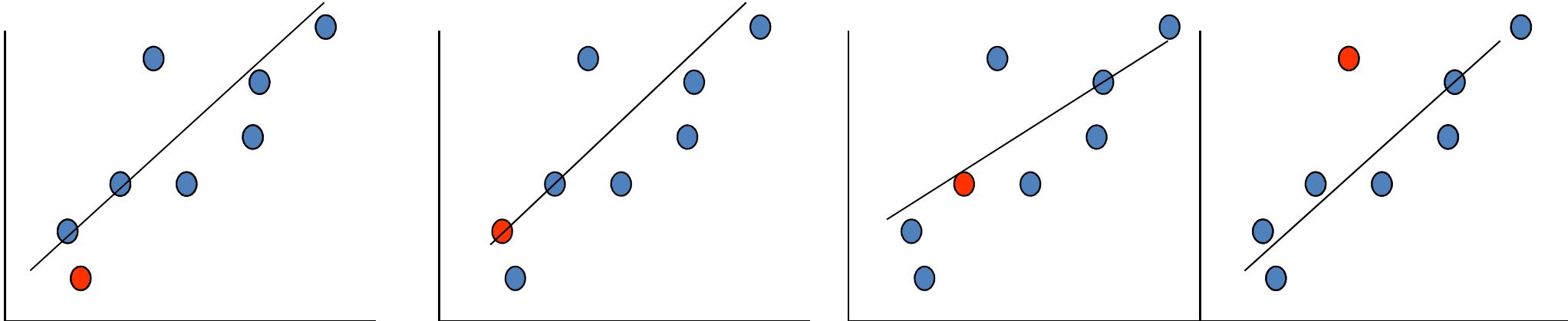
used for calibration

used to calculate prediction error.
Final estimate of MSEP is average of squared errors.

Calculation

OLS
calculation

data used for calibration

estimate of MSEP
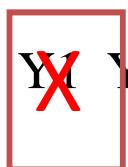
1   ~~Y1~~   Y2   Y3   ……. YN $(Y_1 - \hat{Y}_1(\theta_{-1}))^2$
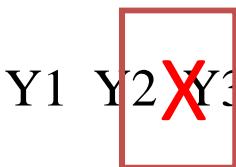
2   Y1   Y2 ~~Y3~~   ……. YN $(Y_2 - \hat{Y}_2(\theta_{-2}))^2$

⋮
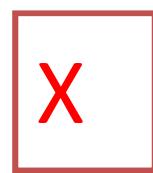
N   Y1   Y2   Y3   ……. YN $(Y_N - \hat{Y}_N(\theta_{-N}))^2$

$$\hat{MSEP} = 1/N \sum \left[Y_i - \hat{Y}_i(\theta_{-i}) \right]^2$$

average to get final
estimate of MSEP

CV requires independence

between errors of data used for calibration and errors of data used for calculating error

– If not, again underestimate MSEP in general

Cross-validation on an hypothetical database…

	Evaluation		Parameterisation

	2004	2005	2006	2007
Poznań (Poland)	Evaluation			
Córdoba (Spain)		Parameterisation		
Worcester (UK)				
Paris (France)				

Cross-validation on an hypothetical database...

	2004	2005	2006	2007
		Evaluation (red)		Parameterisation (cyan)

	2004	2005	2006	2007
Poznań (Poland)		🟥		
Córdoba (Spain)	🟦		🟦	🟦
Worcester (UK)	🟦		🟦	🟦
Paris (France)	🟦		🟦	🟦

Cross-validation on an hypothetical database...

	Evaluation	Parameterisation

	2004	2005	2006	2007
Poznań (Poland)			Evaluation	
Córdoba (Spain)	Parameterisation	Parameterisation		Parameterisation
Worcester (UK)	Parameterisation	Parameterisation		Parameterisation
Paris (France)	Parameterisation	Parameterisation		Parameterisation

Cross-validation on an hypothetical database...

| | Evaluation | Parameterisation |

	2004	2005	2006	2007
Poznań (Poland)				Evaluation
Córdoba (Spain)	Parameterisation			
Worcester (UK)				
Paris (France)				

Cross-validation on an hypothetical database...

	2004	2005	2006	2007
Poznań (Poland)				
Córdoba (Spain)				
Worcester (UK)				
Paris (France)				

Cross-validation on an hypothetical database...

	Evaluation	Parameterisation

	2004	2005	2006	2007
Poznań (Poland)	Parameterisation		Parameterisation	Parameterisation
Córdoba (Spain)		Evaluation		
Worcester (UK)	Parameterisation		Parameterisation	Parameterisation
Paris (France)	Parameterisation		Parameterisation	Parameterisation

Cross-validation on an hypothetical database…

| | Evaluation | Parameterisation |

	2004	2005	2006	2007
Poznań (Poland)	Parameterisation	Parameterisation		Parameterisation
Córdoba (Spain)			Evaluation	
Worcester (UK)	Parameterisation	Parameterisation		Parameterisation
Paris (France)	Parameterisation	Parameterisation		Parameterisation

Cross-validation on an hypothetical database…

	Evaluation	Parameterisation

	2004	2005	2006	2007
Poznań (Poland)	Parameterisation	Parameterisation	Parameterisation	
Córdoba (Spain)				Evaluation
Worcester (UK)	Parameterisation	Parameterisation	Parameterisation	
Paris (France)	Parameterisation	Parameterisation	Parameterisation	

Cross-validation on an hypothetical database...

	2004	2005	2006	2007
Poznań (Poland)				
Córdoba (Spain)				
Worcester (UK)				
Paris (France)				

Cross-validation on an hypothetical database...

	Evaluation	Parameterisation

	2004	2005	2006	2007
Poznań (Poland)	Parameterisation		Parameterisation	Parameterisation
Córdoba (Spain)	Parameterisation		Parameterisation	Parameterisation
Worcester (UK)		Evaluation		
Paris (France)	Parameterisation		Parameterisation	Parameterisation

Cross-validation on an hypothetical database…

	Evaluation	Parameterisation

	2004	2005	2006	2007
Poznań (Poland)	Parameterisation	Parameterisation		Parameterisation
Córdoba (Spain)	Parameterisation	Parameterisation		Parameterisation
Worcester (UK)			Evaluation	
Paris (France)	Parameterisation	Parameterisation		Parameterisation

Cross-validation on an hypothetical database...

	2004	2005	2006	2007
■ Evaluation		**■ Parameterisation**		
Poznań (Poland)				
Córdoba (Spain)				
Worcester (UK)				
Paris (France)				

Cross-validation on an hypothetical database...

 Evaluation   Parameterisation

	2004	2005	2006	2007
Poznań (Poland)				
Córdoba (Spain)				
Worcester (UK)				
Paris (France)				

Cross-validation on an hypothetical database...

	Evaluation	Parameterisation

	2004	2005	2006	2007
Poznań (Poland)	Parameterisation		Parameterisation	Parameterisation
Córdoba (Spain)	Parameterisation		Parameterisation	Parameterisation
Worcester (UK)	Parameterisation		Parameterisation	Parameterisation
Paris (France)		Evaluation		

Cross-validation on an hypothetical database...

	Evaluation	Parameterisation

	2004	2005	2006	2007
Poznań (Poland)				
Córdoba (Spain)				
Worcester (UK)				
Paris (France)				

Cross-validation on an hypothetical database…

	2004	2005	2006	2007
Poznań (Poland)				
Córdoba (Spain)				
Worcester (UK)				
Paris (France)				